Free, forced, and self-excited vibrations

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Rotating motion is a design criterion of rotating machinery, and vibration, as a side effect, diverts energy from this desired condition. There are three major categories of vibrations in mechanical systems: free, forced, and self-excited. These categories are based on fundamental mathematical models of these vibrations, adequate to the observed physical phenomena. The main characteristics of these three vibration categories are shown in Figure 1 and described in Table 1.

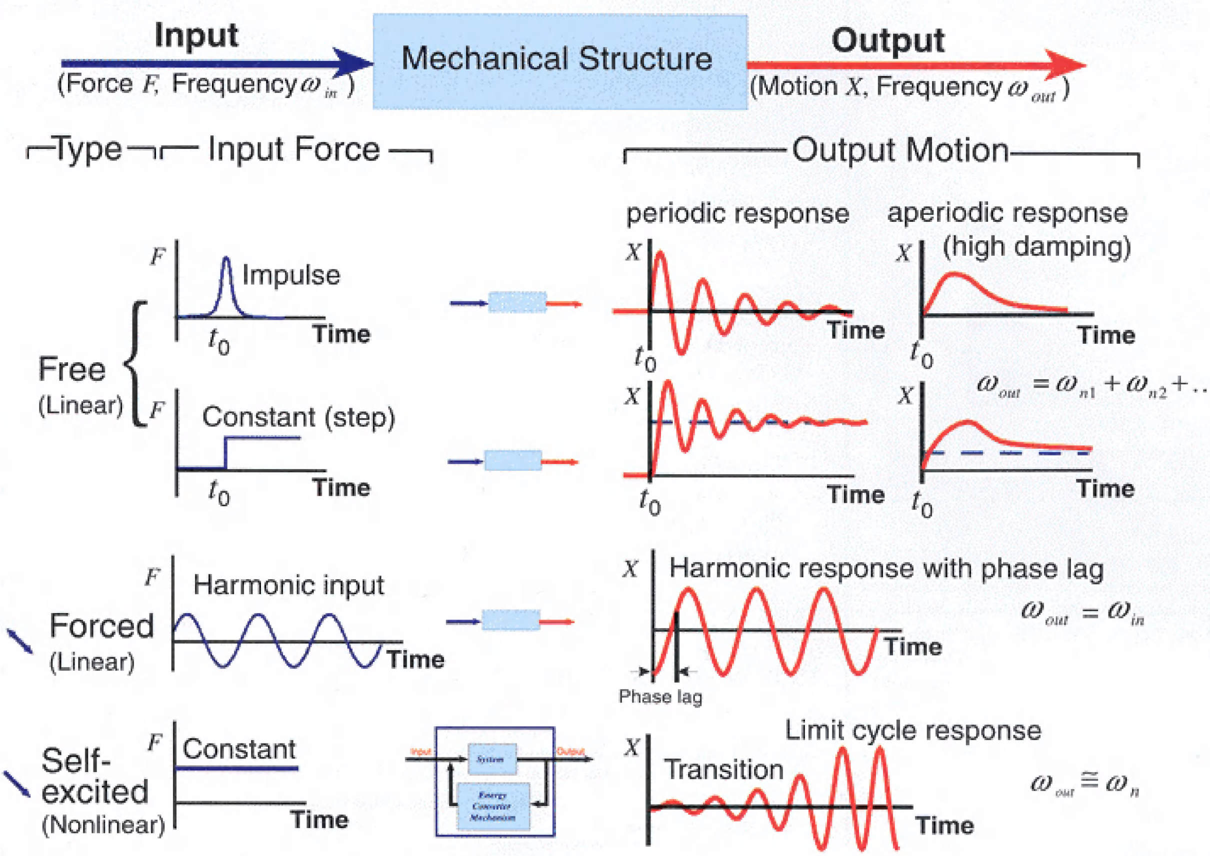

Figure 1. Three categories of vibrations: free, forced, and self-excited, and their characteristic responses. ω_n is a system natural frequency.

The major distinction between free, forced, and self-excited vibrations is the way in which the energy, necessary to sustain vibrations, is provided to the mechanical system.

Free vibrations are excited by an impulse or step force applied to the system and occur with one or several system natural frequencies. In rotating machines, these impulses may be due to an electrical short circuit in a motor or generator, a rub, a surge, a slug of water, a broken blade, etc. If the resulting response is not so high as to cause internal damage, and if the system is stable (has no internal sources of energy equal to or greater than its losses), then the impulse-excited free vibration will decay as the initially input energy dissipates, at a rate which depends on the amount of system damping. The adequate models of the mechanical system free vibrations are usually presented as linear, autonomous (no external, time-dependent excitation) ordinary differential equations.

An unstable mechanical system (containing an internal source of energy greater than its losses) will respond with increasing amplitude vibration. Linear models (see Appendix 1) of such a system predict only the instability threshold, and an unrealistic, infinite growth of vibration amplitudes in the post-instability-threshold domain. Practically, therefore, the linear models become inadequate when the system becomes unstable.

The forced vibrations are excited by continuously applied, time-dependent external forces. Most often these are periodic forces. The most common periodic force in rotating machines is unbalance. Other, nonsynchronous, forces could be caused by vane passing, element passing in rolling element bearings, gear tooth mesh, etc. The mechanical system response follows the excitation, with some phase lag. The frequency of forced vibrations is the same as the excitation frequency (provided that the system is linear). A resonance occurs when an excitation frequency coincides with one of the natural frequencies of the system. For a mechanical resonance, the forced vibration amplitude peaks, and the phase of the exciting force and the lagging vibration response differ by 90 degrees.

The adequate models describing forced vibrations of mechanical systems are linear, non-autonomous (**with** time-dependent excitation), ordinary differential equations. They are similar to free vibration models, except for the addition of the exciting force (the non-autonomous term).

The self-excited vibrations belong to the third category. The energy supply to sustain the periodic vibrations is constant in this case, and comes from a source which may be internal or external to the system.

The system provides the energy dosage from the constant supply through an internal energy converter, as part of a feedback loop, with a frequency characteristic to it; that is, one of the natural frequencies of the system. Self-excited vibrations of machinery rotors can take the form of fluid-induced (oil, steam, or gas) or rotor internal (material) hysteresis-induced whirl or whip. The energy to sustain these vibrations is often drawn from the rotational motion of the rotor.

Adequate models of self-excited vibrations, because they must include nonlinear terms, are nonlinear, ordinary differential equations. Such nonlinear models very often represent the extension of the linear autonomous models for the systems which are potentially unstable, due to the existence of an internal source of energy. The addition of the nonlinear terms to these models elimi-

Type	Characteristics			
	Energy provided by:	Response character:	Response frequency:	Main feature:
Free	Impulse force or sudden change in system element position or velocity.	Transient: periodic or aperiodic, most often decaying (for stable systems).	One or several natural frequencies.	Transient character.
Excited (forced)	Periodic exciting force, external to the system or to the excited mode.	Steady state periodic.	Main frequency is the same as the exciting force frequency.	Creates resonance when exciting force coincides with a system natural frequency.
Self-excited	Constant interactor: external source of constant force or through transfer from another mode.	Transient: periodic with increasing amplitude. Steady state: periodic limit cycle.	Very close to one of the system's natural frequencies.	System nonlinearity required. Feedback loop cycle.

Table 1. Characteristics and features of the three types of vibration

nates the incorrect prediction that, just after the instability threshold, vibration amplitude grows infinitely. As vibration amplitude increases, the nonlinear term becomes dominant and slows down the amplitude growth. The vibration amplitude continues to grow until it reaches the limit cycle of the self-excited vibrations. The self-excited vibrations are then sustained by the constant energy source. The limit cycle is a reflection of the balance between the linear and nonlinear elements in the system.

For fluid whirl or whip, the constant source of energy is shaft rotation, and the energy converter is fluid dragged into circumferential motion by friction.

Free and forced vibrations: linear model

The mathematical model of the one forward lateral mode isotropic rotor is:

$$M\ddot{z} + D_s\dot{z} + Kz + D(\dot{z} - j\lambda\Omega z) = Fe^{j(\omega t+\delta)}$$
$$z = x + jy, \quad j = \sqrt{-1}, \quad {}^{\bullet} = d/dt \tag{1}$$

where $x(t)$, $y(t)$ are rotor lateral displacements lumped into one lateral coordinate $z(t)$; M, D_s, K are rotor modal mass, external damping, and lateral stiffness, respectively; D, λ are fluid environment damping and fluid circumferential average velocity ratio, respectively; Ω is rotor rotative speed, which represents an internal source of energy of the rotor system; F, ω, δ are amplitude, frequency and angular orientation of the nonsynchronously rotating force applied to the rotor; and t is time.

The rotor model (1) is a non-autonomous, linear differential equation with the complex lateral coordinate $z(t)$. If the external exciting force does not exist, (F=0), then the model becomes autonomous.

The autonomous (F=0) equation (1) predicts the threshold of instability [1]

$$\Omega_{st} = \frac{1}{\lambda}\left(1 + \frac{D_s}{D}\right)\sqrt{\frac{K}{M}} \tag{2}$$

where Ω_{st} is the rotative speed at which the free vibrations become harmonic (rotor circular orbit):

$$z(t) = Ce^{j\sqrt{K/M}\,t} \tag{3}$$

with the natural frequency $\sqrt{K/M}$ and a constant amplitude C, as the damping force becomes nullified by the action of the tangential force $jD\lambda\Omega z$. For rotative speeds greater than Ω_{st}, the linear model predicts an exponential growth of free vibration amplitudes:

$$z(t) = Ce^{(\gamma + j\omega_n)t} \tag{4}$$

where ω_n is the "damped" natural frequency of the system, and γ (Appendix 2) is a positive number.

In this situation, the linear model (1) is inadequate to describe the behavior of the rotor; it needs to be complemented by the nonlinear terms which will be discussed in the next section.

So far only the autonomous part of the equation (1) was considered (F=0). Now the concentration will be on forced (or "excited") vibrations of the system, in response to the periodic, nonsynchronous exciting force on the right side of Eq. (1).

The rotor forced response will be periodic with the same frequency as the exciting force:

$$z(t) = Ae^{j(\omega t+\alpha)} \tag{5}$$

where A and α are the response amplitude and phase, respectively.

$$A = \frac{F}{\sqrt{\left(K - M\omega^2\right)^2 + \left[(D + D_s)\omega - D\lambda\Omega\right]^2}} \quad , \tag{6}$$
$$\alpha = \delta - arctan\,\frac{(D + D_s)\omega - D\lambda\Omega}{K - M\omega^2}$$

The peak amplitude, referred to as a resonance, will occur at the frequencies ω at which either the Direct Dynamic Stiffness [1] $K - M\omega^2$ is null (in the case of low damping) or the Quadrature Dynamic Stiffness $(D + D_s)\omega - D\lambda\Omega$ is null (in the case of high damping). In the first case, the resonance is referred to as a "mechanical resonance," and in the second case as a "fluid-induced resonance." In the first case, the resonance frequency is $\omega = \sqrt{K/M}$, in the second case $\omega = \lambda\Omega/(1+D_s/D)$, both of which represent rotor natural frequencies.

Self-excited vibration: nonlinear model

The nonlinear model of the one forward lateral mode isotropic rotor results from Eq. (1) by the addition of a nonlinear term. For simplicity, assume that only the rotor radial stiffness is nonlinear, and take into account only the first term of the power series of the isotropic nonlinear stiffness function [2]. The rotor nonlinear model is, therefore, as follows:

$$M\ddot{z} + D_s\dot{z} + Kz + D(\dot{z} - j\lambda\Omega z) + K_1|z|^2 z = 0 \quad , \tag{7}$$
$$|z| = \sqrt{x^2 + y^2}$$

where K_1 is the coefficient of the nonlinear stiffness term. The external exciting force is omitted in Eq. (7) for clarity, resulting in an autonomous system.

It will be shown below that Eq. (7) has an exact solution describing the limit cycle self-excited vibrations (Figure 2):

$$z(t) = Be^{j\omega_s t} \tag{8}$$

where ω_s, B are the frequency and amplitude of the self-excited limit cycle, which can be calculated when Eq. (8) is substituted into Eq. (7):

$$\left[-M\omega_s^2 + jD_s\omega_s + K + jD(\omega_s - \lambda\Omega) + K_1B^2\right]Be^{j\omega_s t} = 0 \tag{9}$$

To satisfy this equation, the expression inside the brackets must equal zero. Splitting it into real and imaginary parts, the unknown frequency ω_s and amplitude B can be calculated:

$$\omega_s = \frac{\lambda\Omega}{1 + D_s/D}, \qquad B = \sqrt{\frac{1}{K_1}\left(\frac{M\lambda^2\Omega^2}{(1 + D_s/D)^2} - K\right)} \tag{10}$$

The transient process, which starts at the instability threshold and ends at the limit cycle, can be calculated when the solution of Eq. (7) is assumed as follows [3]:

$$z = \psi(t)e^{j\omega_t t} \tag{11}$$

where $\psi(t)$ represents a variable response amplitude, and ω_t is the frequency at the instability threshold. For the model (7) $\omega_t = \sqrt{K/M}$.

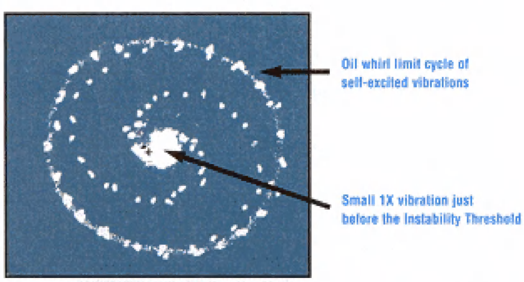

Figure 2. Self-excited vibration transient process: increasing rotor vibration amplitude leading to the limit cycle of oil whirl. Each white dot in this oscilloscope picture corresponds to one rotor rotation.

References

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Appendix 1

Linear terms in an ordinary differential equation contain only terms with the first powers of acceleration ($\ddot{z}$) velocity ($\dot{z}$), and displacement (z) with some constant (or time t dependent) coefficients. If z, $\dot{z}$, $\ddot{z}$ appear in the equation as functions, higher powers, or products (for example, $\sin(z)$, z^3, or $\dot{z}z$), the differential equation becomes nonlinear.

Appendix 2

The solution of the eigenvalue problem for Eq. (1) provides two eigenvalues $s_{1,2}$ [1]:

$$s_{1,2} = -\frac{D_s + D}{2M} \pm \frac{1}{\sqrt{2}}\sqrt{-E + \sqrt{E^2 + \left(\frac{D\lambda\Omega}{M}\right)^2}} + j\omega_n \tag{A1}$$

where ω_n is the "damped" natural frequency,

$$\omega_n = \frac{1}{\sqrt{2}}\sqrt{E + \sqrt{E^2 + \left(\frac{D\lambda\Omega}{M}\right)^2}},$$

$$E = \frac{K}{M} - \left(\frac{D + D_s}{2M}\right)^2$$

and the real part of one eigenvalue (A1) is always negative.

The instability threshold occurs when the real part of the other eigenvalue (A1) becomes zero. The instability condition (2) results from the solution of the second real part set equal to zero:

$$-\frac{D_s + D}{2M} + \frac{1}{\sqrt{2}}\sqrt{-E + \sqrt{E^2 + \left(\frac{D\lambda\Omega}{M}\right)}} = 0$$

At the instability threshold, the natural frequency becomes "undamped" and is equal to $\sqrt{K/M}$. For the post-instability-threshold rotative speeds, the real part of one eigenvalue becomes positive (see Eq. (4)):

$$\gamma = -\frac{D_s + D}{2M} + \frac{1}{\sqrt{2}}\sqrt{-E + \sqrt{E^2 + \left(\frac{D\lambda\Omega}{M}\right)^2}} > 0$$

as the term $(D\lambda\Omega/M)$ becomes dominant.